21p.

[redacted]

N64-20733

CAT23 CODE-1

NASA CR-53605

[redacted]

COUPLING OF SOUND AND PANEL VIBRATION BELOW THE CRITICAL FREQUENCY

(NASA Contract

P. W. Smith, Jr.
Bolt, Beranek and Newman, Inc.,
Cambridge, ~~38~~, Massachusetts

04 31643

20733

ABSTRACT

Prior analysis has shown that the sound power radiated from reverberant vibration of a simply supported panel is proportional to the length of the panel's perimeter when the frequency is below the critical frequency. A simple physical interpretation was made in terms of the mode shape of resonant modes. Other analyses for power radiation from a single straight boundary on an infinite panel indicated that twice as much power radiates from a clamped edge as from the simply supported edge, the spatial mean square velocity being held constant.

It is shown here that the increased radiation from clamped edges is also predictable from simple considerations of the mode shape. Moreover, among all boundaries having a purely reactive rotatory impedance and restricting transverse displacement to zero, the clamped edge is found to be the most efficient radiator. For some impedances, power radiation is found to approach zero. AUTHOR

2

[1962]

OTS PRICE

XEROX $ 2.60 ph

MICROFILM $ none

Submit

→ Inc.

[redacted]

SIMPLY SUPPORTED PANEL

In his study of the coupling between sound waves in a fluid and bending vibrations of a simply supported rectangular panel surrounded by an infinite rigid baffle, Maidanik concluded that the strength of coupling is proportional to the length of the panel's perimeter when the frequency is less than the critical frequency.[1] (The critical frequency is that at which the wavelength of free straight-crested bending waves in an infinite panel of the same thickness would equal the wavelength of free plane waves in the unobstructed fluid.) His measure of coupling is the radiation resistance, defined in every case as the time average of radiated sound power for a unit value of the space and time average of the square of vibrational velocity. However, one may show by reciprocity arguments that the same conclusion holds when a diffuse field of sound waves forces the panel.[2] The factors of proportionality in Maidanik's relation involve no parameters of the panel except the ratio of frequency to critical frequency. The relation is restricted to situations in which resonant vibration predominates.

Maidanik has also given a simply physical interpretation of the relation of coupling to the perimeter, which is based on considerations of the characteristic functions, or mode shapes, of the natural modes. A review of this phase of his

[redacted]

work will form the basis for the present extension. For a rectangular panel with simply supported edges of lengths ℓ_x and ℓ_y in the x and y directions, the characteristic functions are

$$\begin{aligned}
\psi_M &= (\sqrt{2} \sin k_{Mx} x)(\sqrt{2} \sin k_{My} y) \quad , \\
& k_{Mx}\ell_x/\pi = m_x \quad \text{, an integer} \quad , \\
& k_{My}\ell_y/\pi = m_y \quad \text{, an integer} \quad , \\
& k_{Mx}^2 + k_{My}^2 = k_M^2 \quad .
\end{aligned} \tag{1}$$

At resonance, k_M is equal to k_p, the wavenumber for free straight-crested waves on the infinite panel. Within the confines of the panel, $0<x<\ell_x$ and $0<y<\ell_y$, the modal displacement is identical to the superposition of four straight-crested waves. These consist of the wave with a vector wavenumber $\underline{k}_M$ having components k_{Mx} and k_{My}, and of its reflections in the x and y axes. Equation 1 has been normalized so that the spatial mean square value of ψ_M is unity.

We consider a frequency well below the critical frequency, so that $k_p^2 \gg k^2$ where k is the wavenumber of free plane waves in the fluid. The various modes resonant at about the same frequency, i.e. with about the same k_M, have different directions, ranging from nearly normal to the x edges when k_{Mx} is small to nearly normal to the y edges. Maidanik showed that only those modes which have directions nearly normal to one of the edges are well coupled. Specifically, good coupling requires that either k_{Mx} or k_{My} be less than k.

Consider a mode nearly normal to the x edges, for which case

$$
\begin{aligned}
& k_{Mx} < k \\
& k_{My}^2 = k_p^2 - k_{Mx}^2 \approx k_p^2 \gg k^2 \quad .
\end{aligned}
\tag{2}
$$

This mode's shape, ψ_M, has a wavelength in the y direction which is very short compared with the sound wavelength. A section of the vibrating panel, taken parallel to the y edges, is shown in Fig. 1. Since the wavelength of the disturbance is small compared with the sound wavelength, the radiation from one crest-to-node segment, such as A, is effectively cancelled by the radiation from the adjacent segment, B, which is out of phase with it. By extending this argument, one is led to conclude that all the radiation from the lightly shaded central portion is effectively cancelled, so that the radiation from the section as a whole must be accounted for by the radiation from the heavily shaded end segments, lying between the panel's edges and the nearest crests.

The approximation employed here is similar to the well-known use of "volume velocity" in calculating the radiation from a vibrating source all of whose dimensions are small in comparison to the wavelength of sound. The volume velocity is proportional to the net area under the whole deflection curve. In contrast thereto, the present approximation is applicable to a source whose size may be large in comparison to the wavelength, so long as the vibrational wavelength is small. When the size of the source is small, the two approximations are identical.

In the x direction, the shape of a mode nearly normal to the x edges has a wavelength longer than the sound wavelength. Therefore cancellation does not take place along this direction. The conclusion is that sound radiation from the modal vibration of the whole panel is identical with the radiation from a pair of narrow bars of length ℓ_x, located on the x edges of the panel and vibrating with mode shapes

$$\phi = \sqrt{2} \sin k_{Mx} x \quad , \quad k_{Mx} < k \quad . \tag{3}$$

The spatial mean square velocity of each bar equals the spatial mean square velocity of the panel as a whole. The effective width of each bar is the heavily shaded area in Fig. 1, i.e., the area from panel edge to nearest crest under the curve

$$\sqrt{2} \sin k_{My} y \quad .$$

The effective width equals

$$w_e = \sqrt{2}/k_{My} \approx \sqrt{2}/k_p \quad . \tag{4}$$

Each equivalent bar satisfies the conditions of a bar vibrating above its critical frequency, for which the radiation resistance is known to equal

$$R_{rad} = \frac{1}{2}\rho_o \, c \, k \, w_e^2 \, \ell_x \tag{5}$$

where ρ_o is the density and c the sound speed of the fluid.[3] This result is independent of mode number and proportional to ℓ_x for every mode nearly normal to the x edges (i.e. with $k_{Mx}<k$). A similar result, proportional to ℓ_y, can be obtained for every mode nearly normal to the y edges. When it has been confirmed that equal numbers of modes of the two types are resonant in a given small interval of frequency, the observed proportionality is found between the perimeter $P=2(\ell_x+\ell_y)$ and the total radiation resistance for reverberant panel vibration, in which modes of all types have equal energy. The analytical result is

$$R_{rad} = \rho_o c \; P \; 2k^2/\pi k_p^3 \quad . \tag{6}$$

Maidanik has presented this physical interpretation and confirmed that the radiation resistance predicted therefrom is identical with the leading term of an expansion of analytical results in powers of the small quantity k/k_p (ref. 1, Eq. 2.39a).

Indeed, more detailed consideration shows that all the leading characteristics of the radiated sound follow from such an interpretation--even the directivity functions for individual modes. The sound pressure radiated to a great distance in a direction specified by the unit vector $\underline{\Omega}$ is proportional to a surface integral over the panel:

$$p(\underline{\Omega}) \propto \int e^{-ik\underline{\Omega}\cdot\underline{r}}\, \psi_M(\underline{r})\; d\sigma \tag{7}$$

where $\underline{r}$ is a position vector in the plane of the panel. (Here the exponential is the only variable part of the Green's function for the point $\underline{r}$ on the panel and a distant observation point.) In combination with the mode shape specified in Eq. 1, this equation leads to a product of two line integrals of the form

$$I_y = \sqrt{2}\int e^{-iky\underline{\Omega}\cdot\underline{e}_y} \sin k_{My}y\; dy \quad , \tag{8}$$

where $\underline{e}_y$ is the unit vector along the y edge of the panel. When $k_{My}^2 \approx k_p^2 >> k^2$, the cancellation central to the physical interpretation is reflected mathematically by the limiting value of the integral:

$$I_y \to w_e(1 \pm e^{-ik\ell_y \underline{\Omega} \cdot \underline{e}_y}) \text{ as } k/k_{My} \to 0 \quad , \tag{9}$$

where w_e is the effective width given in Eq. 4; the plus sign is to be chosen when the end segments of ψ_M have the same sign (i.e., when the mode integer m_y is odd) and vice versa.

The second line integral in Eq. 7, I_x, gives the pressure radiated by a strip, of unit width, parallel to the x edge and vibrating with the mode shape specified in Eq. 3. The formula is identical with that for the pressure radiated from a bar of unit width vibrating at a frequency <u>above</u> its critical frequency. Since $k_{Mx}<k$, there are angles $\underline{\Omega}$ at which every part of the bar contributes equally to the radiation; at other angles effectively complete destructive interference occurs. This strip, or bar, behaves as a directive line array, and the sound power flux is distributed along its whole length. This conclusion will be important in our latter assessment of the effects of boundary conditions.

OTHER EDGE CONDITIONS

This interpretation of the sound radiation from a panel as being due to the vibrating strips along its edges, when the frequency is less than critical, has been restricted to panels with simply supported edges. Maidanik[1] and Lyon[4] extended the analysis to other boundary conditions by analytical flanking attacks.

On Lyon's suggestion, both he and Maidanik investigated the ratio of radiation resistance to perimeter by considering a truly diffuse, reverberant field of straight-crested waves incident upon an infinite straight boundary in an infinite plane panel. In essence, this procedure replaces the quasi-diffuse reverberant field of the finite large panel, which has discrete angles of incidence distributed uniformly in angle, by an infinitely dense uniform distribution of angles of incidence. Since the discrete angles of incidence of the finite panel fluctuate with frequency variations, the procedure should yield a good average estimate.

One possible reservation of confidence is readily answered in a qualitative way. Granted that, according to Maidanik's physical interpretation, the sound radiation may be accounted for by the strip nearest the boundary, it is still conceivable that this radiation could be significantly affected by the boundary conditions at the end of the strip, i.e. at the corners of the panel. However such effects are not to be expected because, as observed above, the power flux is distributed uniformly along the strip. Small differences in the mode shape near the corners, such as between clamped and simply supported edges, can have little effect on the total flux. On the other hand, differences in boundary conditions along the edge of the strip may lead to significant effects through modification of the effective width (Eq. 4).

Both Maidanik[1] and Lyon[4] confirmed that his analysis for an infinitely long, straight edge yielded the same result as the modal analysis (Eq. 6) in the case of a simply supported edge that transmits no energy. Each also concluded that exactly twice as much sound power is radiated from a clamped boundary, insofar as leading terms in (k/k_p) are concerned. In all cases, the important contributions to sound power are those from waves incident upon the boundary at an angle near the normal.

We wish to demonstrate that the increased radiation for a clamped edge is also derivable from simple considerations of the mode shape, similar to Maidanik's physical interpretation for a simply supported edge. We shall investigate as well the whole class of boundaries for which (1) there is no transverse displacement at the edge, (2) no vibration is transmitted across the edge, and (3) the ratio of slope to curvature of the vibrating panel, evaluated normal to the edge, is an arbitrary constant parameter. In conjunction, the last two boundary conditions are equivalent to specifying a rotatory impedance whose real part vanishes and whose imaginary part is an arbitrary constant. (The ratio of slope to curvature must be real if no energy is transmitted.)

Maidanik and Lyon found that angles of incidence near normal were most important. Here we consider a bending wave in an infinite panel which is incident precisely normally upon a straight boundary at $x=0$. The boundary conditions to be considered are given above; vibration is restricted to positive values of x. The general solution for finite transverse deflection is

$$\psi(x) = \sqrt{2}\cos(k_p x + \phi) - \sqrt{2}\cos\phi\, e^{-k_p x} . \qquad (10)$$

The mean square value of ψ for $x>0$ has been adjusted to unity. The slope, curvature, and rotatory reactance X at the boundary are determined by the parameter ϕ:

$$\begin{aligned} \psi'(0) &= -2\,k_p \sin(\phi - \tfrac{1}{4}\pi) \\ \psi''(0) &= -2\sqrt{2}\,k_p^2\cos\phi \qquad (11) \\ X \equiv -D\psi''(0)/\omega\psi'(0) &= (2\omega m/k_p^3)/(1-\tan\phi) , \end{aligned}$$

where D is the panel's bending rigidity, ω is frequency, m is the panel's mass per unit area, and $k_p^4 = \omega^2 m/D$. The implicit time dependence is $\exp(i\omega t)$, and positive values of X imply a massive boundary. Without loss of generality, ϕ will be restricted to the range 0 to π. Note the particular cases of the simply supported edge, $\phi = \frac{1}{2}\pi$, and the clamped edge, $\phi = \frac{1}{4}\pi$. Curves of deflection $\psi(x)$ near the edge are given in Fig. 2 for several values of ϕ.

For values of $k_p x$ greater than about 2π, the exponential term in Eq. 10 is quite negligible and ψ is practically sinusoidal. In that region, Maidanik's physical argument of detailed cancellation of sound radiation (for $k<k_p$) is equally valid for all values of ϕ. In generalization of Maidanik's interpretation, we might define as the equivalent width w_e of the strip at the boundary, the <u>net</u> area under the $\psi(x)$ curve from the boundary to the crest at $k_p x \approx 2\pi-\phi$, beyond which cancellation takes place. However, for analytical convenience, we use the infinite integral:[5]

$$w_e \equiv \lim_{n\to\infty} \left| \int_0^{(n\pi-\phi)/k_p} \psi(x)dx \right| = (2/k_p)\left|\sin(\phi + \tfrac{1}{4}\pi)\right|. \tag{12}$$

The sound power radiated from the boundary is proportional to w_e^2 (Eq. 5). Curves proportional to w_e^2 and to the rotatory reactance X of the boundary are given in Fig. 3. This generalized physical interpretation correctly accounts for the sound radiation from a simply supported edge (Eq. 4), and also correctly predicts that twice as much power should be radiated from the clamped edge.

Two other conclusions should be noted. First, the clamped edge leads to the largest radiation resistance of any edge with purely reactive rotatory impedance and with zero transverse displacement. Secondly, there is a massive rotatory reactance,

$$X = \omega m/k_p^3 \quad ,$$

for which the equivalent width vanishes. This is the case for $\phi = \frac{3}{4}\pi$; the corresponding deflection curve is plotted in Fig. 2C. One should not conclude that the radiation resistance really vanishes exactly in this case, but rather that it is dependent on residual, less efficient, coupling. Moreover, the complications resulting from non-vanishing of the transverse displacement at $x=0$ and from finite energy transmission, which are usual for real boundaries and are encompassed by Lyon's general analysis,[4] are not accounted for here.

One should note that the present analysis of the effect of boundary conditions on radiation resistance is also directly applicable to a beam vibrating at frequencies below its critical frequency. (The beam with simply supported ends was analyzed in reference 3.) For such a beam, the net coupling to sound is attributable to the two end regions as shown in Fig. 1; it is numerically equal to the coupling to a pair of rigid pistons located at the ends, each having an area equal to the product of the beam's width by the equivalent width (Eq. 12), and each having a mean square velocity equal to the mean square velocity of the whole beam.

MODES OF SMALL ORDER NUMBER

The present analysis for vibrations of a semi-infinite beam or panel is not appropriate to the first few modes of a finite structure. Nonetheless, it can be shown that many of the conclusions are still valid, even for the fundamental mode.

Consider the lower modes of a finite beam. The results for it should also be applicable to a long narrow panel, but not to a squarish panel which is "all corners". The exponential term in the solution for the infinite structure dies out so fast with distance from the boundary that the difference between the deflection curves of the semi-infinite beam and the finite beam having the same value of k_p is quite small, even for the second mode (see Fig. 2). Therefore the results of the present analysis, and the expression for "equivalent width" (Eq. 12), are applicable to all modes except possibly the fundamental.

Let us examine the fundamental modes of the finite beam in the two cases of clamped and simply supported ends. Note in the case of the fundamental modes that the central region of cancellation is non-existent, so that the net coupling to sound is determined by the total area under the deflection curve; in the present notation this area is equated to $2w_e$.

The expressions for w_e given in Eqs. 4 and 12 for simply supported ends are precisely correct. For the fundamental mode with clamped ends it can be shown that

$$w_e = 1.965/k_p \quad ,$$

which is only 1.8 percent smaller than the expression in Eq. 12, derived for the semi-infinite structure.[6] Therefore Eq. 12 is an excellent approximation for all modes, for clamped and supported ends.

The bald conclusion that "coupling of sound to structure with clamped edges is twice that for simply supported edges, even in the fundamental mode" can be applied if certain precautions are observed. First, the spatial mean square deflections must be equal in the two cases. Second, the conclusion refers only to the analytic expression of coupling as a function of k and k_p. Two identical structures vibrating in the same mode with different boundary conditions will resonate at different frequencies and at different values of k and k_p. These differences are greater for modes of lower order number. Third, the fundamental mode of a nearly square panel remains to be investigated.

(This work was sponsored by the National Aeronautics and Space Administration.)

REFERENCES

1. G. Maidanik, J. Acoust. Soc. Am. <u>34</u>, 809-826 (1962).

2. P. W. Smith, Jr., J. Acoust. Soc. Am. <u>34</u>, 640-647 (1962).

3. R. H. Lyon and G. Maidanik, J. Acoust. Soc. Am. <u>34</u>, 623-639 (1962).

4. R. H. Lyon, J. Acoust. Soc. Am. <u>34</u>, 1265-1268 (1962).

5. It is probable that this infinite integral is the precise expression for equivalent width in the limit as frequency ω and k approach zero. Compare the familiar use of net volume velocity in calculating the low frequency radiation from a piston. Then this discussion identifies the net area as principally localized in the strip out to the second crest, from which conclusion is derivable a criterion for using the low frequency approximation: that $(2\pi-\phi)k/k_p$ be small.

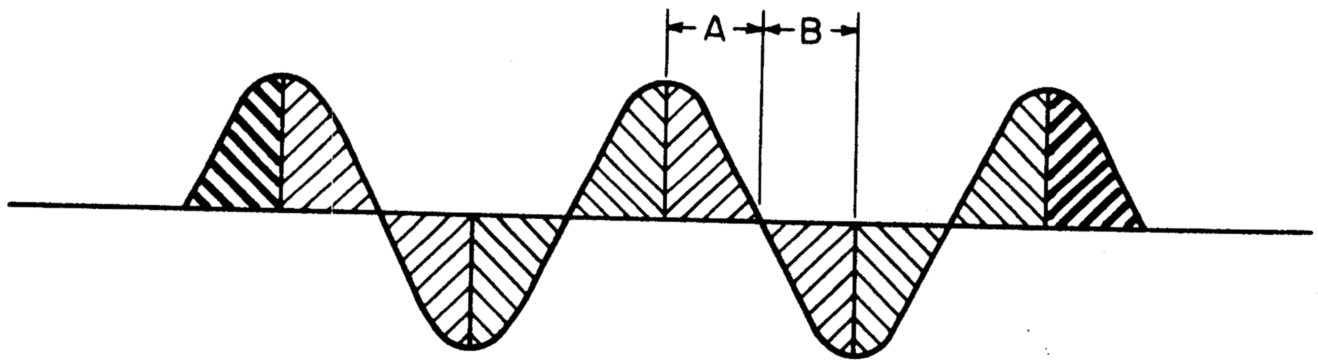
6. The integral of deflection is related to the values at the ends of the third derivative, because ψ satisfies the fourth-order bending wave equation. The values of the third derivative were taken from R.E.D. Bishop and D. C. Johnson, <u>Vibration Analysis Tables</u> (The University Press, Cambridge, England, 1956).

CAPTIONS

Fig. 1 Sinusoidal deflection curve for simply supported panel. At low frequencies, coupling to sound due to the lightly shaded, central part cancels, so that the net coupling is due to the heavily shaded edge segments.

Fig. 2 Deflection curves for bending waves satisfying rotatory reactance boundary conditions at $x=0$. The value of the reactance is determined by the parameter ϕ.

Fig. 3 Variation with reactance parameter ϕ of the sound power radiation, proportional to w_e^2, for various boundary reactances, proportional to $(1 - \tan\phi)^{-1}$.


A
B

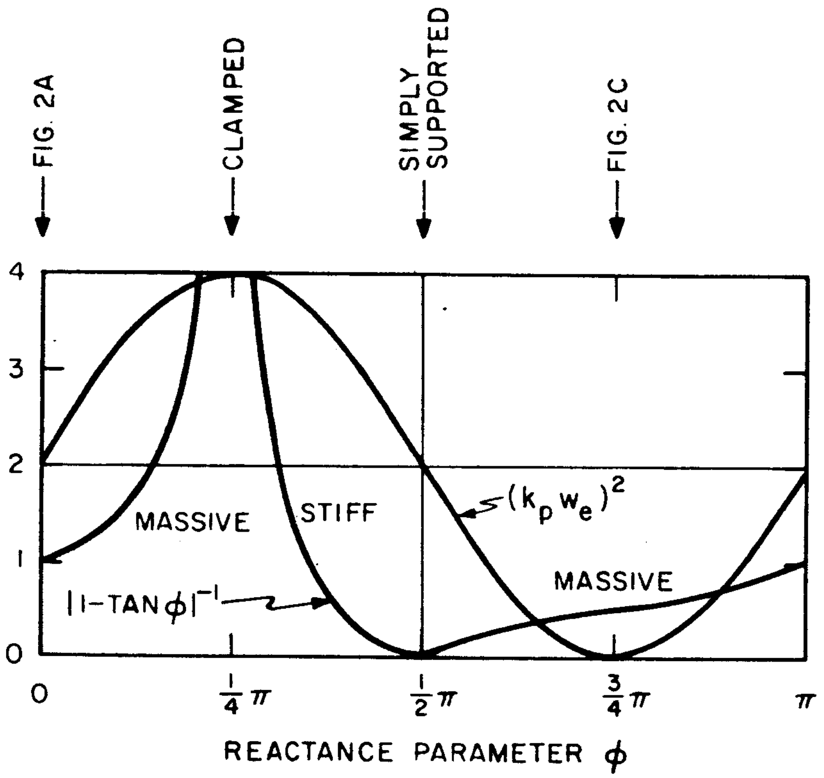
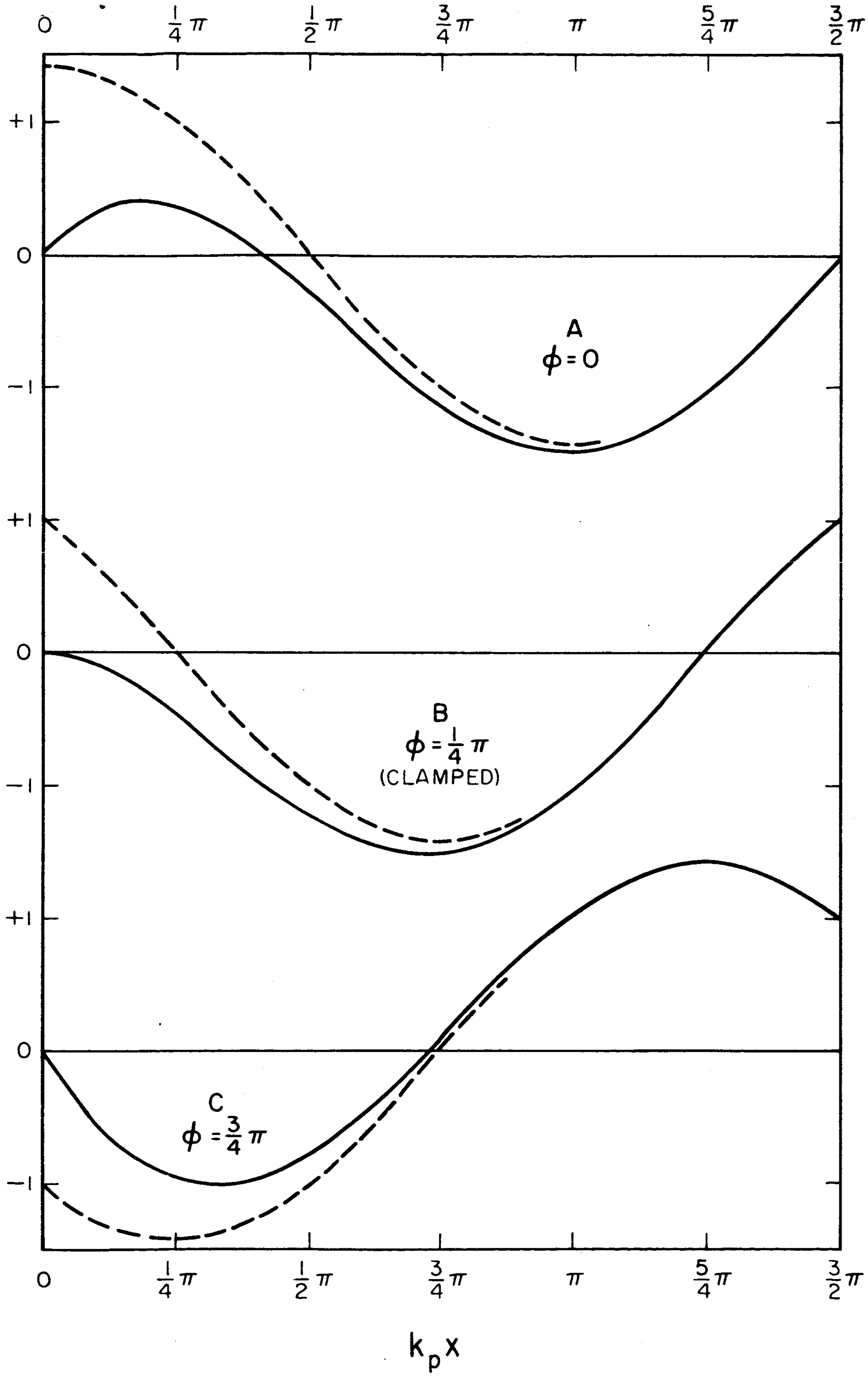


FIG. 2A
CLAMPED
SIMPLY SUPPORTED
FIG. 2C
4
3
2
1
0
MASSIVE
STIFF
MASSIVE
$(k_p w_e)^2$
$|1-\tan\phi|^{-1}$
0
$\frac{1}{4}\pi$
$\frac{1}{2}\pi$
$\frac{3}{4}\pi$
π
REACTANCE PARAMETER ϕ



0
$\frac{1}{4}\pi$
$\frac{1}{2}\pi$
$\frac{3}{4}\pi$
π
$\frac{5}{4}\pi$
$\frac{3}{2}\pi$
+1
0
-1
A
$\phi = 0$
B
$\phi = \frac{1}{4}\pi$
(CLAMPED)
C
$\phi = \frac{3}{4}\pi$
$k_p x$